FINAL TERM EXAMINATION 2017-18
MATHEMATICS ANSWER KEY - CLASS XI

SNO	SECTION-A	MKS
1.	$a=5$ or $a=-3$	1
2.	$-2\sqrt{3}+2i$	or
3.	101	0
4.	Exclusive or	

SECTION - B

5. $a+ib = \dfrac{z_1 z_2}{\bar{z}_1} = \dfrac{(1+i)(-2+4i)}{1-i}$

$= \dfrac{-6+2i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{-8-4i}{2}$

$\therefore a=-4, \ b=-2$

6. $B \cup C = \{1,3,4,5,6\}$ $A \cap (B \cup C) = \{1,3,4\}$

$A \cap B = \{3,4\}$

$A \cap C = \{1,3\}$ $\therefore (A \cap B) \cup (A \cap C) = \{1,3,4\}$

7. $a_p = q \Rightarrow a+(p-1)d = q \longrightarrow ①$

$a_q = p \Rightarrow a+(q-1)d = p \longrightarrow ②$

$① - ② \Rightarrow (p-q)d = q-p \Rightarrow d=-1$

From ①, $a = p+q-1$

$\therefore a_n = (p+q-1)+(n-1)(-1) = \underline{p+q-n}$

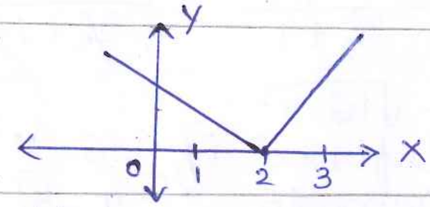
8. Eqn of line is $\dfrac{x}{a}+\dfrac{y}{b}=1$

Given, $b=2a$. The line passes through $(4,-2)$.

$\Rightarrow \dfrac{4}{a}+\dfrac{-2}{2a}=1 \Rightarrow \dfrac{3}{a}=1 \Rightarrow a=3, \ b=6$

$\therefore$ Eqn $\rightarrow \dfrac{x}{3}+\dfrac{y}{6}=1 \Rightarrow \underline{2x+y=6}$

9. $f(x) = \begin{cases} x-2, & \text{if } x>2 \\ 0, & \text{if } x=2 \\ 2-x, & \text{if } x<2 \end{cases}$



10. Focus $(0,-3)$ lies on negative Y-axis.

$\therefore$ Eqn of parabola is $x^2=-4ay$, where $a=3$

$\Rightarrow \underline{x^2=-12y}$

SNO		MKS

11. P(fruit is an apple) $= \frac{20}{30}$

P(it is a good fruit) $= \frac{22}{30}$

$\therefore$ P(either an apple or a good fruit) $= \frac{20+22-15}{30} = \frac{\cancel{27}^{9}}{\cancel{30}_{10}}$

12. Converse :- If the ratio of corresponding sides of 2 triangles are equal, then the triangles are similar.

Contrapositive :- If the ratio of corresponding sides of two triangles are not equal, then they are not similar.

SECTION – C

13. Let $P(n): \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$

$P(1)$: LHS $\rightarrow \frac{1}{1\cdot4} = \frac{1}{4}$, RHS $\rightarrow \frac{1}{3(1)+1} = \frac{1}{4}$

$\therefore P(1)$ is true.

Let $P(K)$ be true $\Rightarrow P(K): \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \dots + \frac{1}{(3K-2)(3K+1)} = \frac{K}{3K+1} \rightarrow (1)$

Now to prove, $P(K+1)$ is true.

$P(K+1): \frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \dots + \frac{1}{(3K-2)(3K+1)} + \frac{1}{(3K+1)(3K+4)}$

$= \frac{K}{3K+1} + \frac{1}{(3K+1)(3K+4)}$ (from (1))

$= \frac{1}{3K+1}\left[K + \frac{1}{3K+4}\right]$

$= \frac{1}{\cancel{3K+1}}\left[\frac{\cancel{(3K+1)}(K+1)}{3K+4}\right] = \frac{K+1}{3K+4}$

conclusion.

14. Let a point $P(0, y, z)$ in the YZ-plane divide $A(3, 5, -7)$ & $B(-2, 1, 8)$ in the ratio $K:1$.

$\therefore$ coordinates of $P(0, y, z) = \left(\frac{3-2K}{K+1}, \frac{5+K}{K+1}, \frac{-7+8K}{K+1}\right)$

$\Rightarrow \frac{3-2K}{K+1} = 0 \Rightarrow K = 3/2$

$\therefore P = \left(\frac{3-2(3/2)}{3/2+1}, \frac{5+3/2}{3/2+1}, \frac{-7+8(3/2)}{3/2+1}\right) = \left(0, \frac{13}{5}, 2\right)$

15. $f(x) = \sqrt{16-x^2}$

f is defined if $16 - x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow x \in [-4, 4]$

$\therefore$ Domain $= [-4, 4]$

Now, let $y = \sqrt{16-x^2} \Rightarrow y^2 = 16 - x^2 \Rightarrow x = \sqrt{16-y^2}$

'x' is defined if $16 - y^2 \geq 0 \Rightarrow y^2 \leq 16 \Rightarrow y \in [-4, 4] \rightarrow (1)$

But $y = \sqrt{16-x^2} \Rightarrow y \geq 0 \rightarrow (2)$

$\therefore$ from (1) & (2), we get Range $= [0, 4]$.

(OR)

$f(x) = \sqrt{x-[x]}$. f is defined if $x-[x] \geq 0$

we know that $0 \leq x-[x] \leq 1$ for all $x \in R$.

$\therefore$ Domain $= R$.

we have $0 \leq x-[x] \leq 1$ for all $x \in R$

$$\Rightarrow 0 < \sqrt{x-[x]} < 1$$

$$\Rightarrow 0 < f(x) < 1$$

$\therefore$ Range $= (0,1)$.

16. $4\sin\alpha \cdot \sin\left(\alpha+\frac{\pi}{3}\right) \cdot \sin\left(\alpha+\frac{2\pi}{3}\right)$

$= 2\sin\alpha \cdot 2\sin\left(\alpha+\frac{\pi}{3}\right) \cdot \sin\left(\alpha+\frac{2\pi}{3}\right)$

$= 2\sin\alpha \left[\cos\left(-\frac{\pi}{3}\right) - \cos(\pi+2\alpha)\right]$

$= 2\sin\alpha \left[\frac{1}{2} + \cos 2\alpha\right]$

$= \sin\alpha + 2\sin\alpha(1-2\sin^2\alpha)$

$= \sin\alpha + 2\sin\alpha - 4\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha = \underline{\sin 3\alpha}$

17. $Z = \frac{5-i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{13+13i}{4+9} = \frac{13+13i}{13} = 1+i$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$ $\quad \therefore Z = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

18. $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\pi/2}$ $\left(\frac{0}{0}\text{ form}\right)$

Let $y = x-\frac{\pi}{2}$. As $x \to \frac{\pi}{2}$, $y \to 0$

$\therefore \lim_{y\to 0} \frac{\tan 2\left(\frac{\pi}{2}+y\right)}{y} = \lim_{y\to 0} \frac{\tan(\pi+2y)}{y}$

$= \lim_{y\to 0} \frac{\tan 2y \times 2}{y \times 2} = 1 \times 2 = \underline{2}$

(OR)

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} (x^3 - 3x+7) = -1+3+7 = \underline{9}$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} (3x+k) = -3+k$

Since $\lim_{x\to -1} f(x)$ exists (given)

$\Rightarrow LHL = RHL \Rightarrow 9 = -3+K \Rightarrow K = 12$.

19. $f(x) = x\sin x$

$f(x+h) = (x+h)\sin(x+h)$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x\sin(x+h) + h\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h\to 0} \frac{x[\sin(x+h) - \sin x] + h\sin(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{x\left[2\cos\left(\frac{2x+h}{2}\right)\cdot \sin(h/2)\right]}{h} + \frac{\cancel{h}\sin(x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} x\left[\cos\left(\frac{2x+h}{2}\right)\cdot \frac{\sin h/2}{h/2}\right] + \sin(x+h)$$

$$= \lim_{h\to 0} x\cos\left(\frac{2x+h}{2}\right) \cdot \lim_{h/2\to 0} \frac{\sin h/2}{h/2} + \lim_{h\to 0}\sin(x+h)$$

$$= x\cos x + \sin x$$

20. No. of ways of choosing 10 questions $= ({}^6C_4 \times {}^7C_6) + ({}^6C_5 \times {}^7C_5) + ({}^6C_6 \times {}^7C_4)$

Part A(6)	Part B(7)
4	6
5	5
6	4

$= \left(\frac{\cancel{6}^3 \times 5}{\cancel{2}} \times 7\right) + \left(\cancel{6}^3 \times \frac{7\times 6}{\cancel{2}}\right) + \left(1 \times \frac{7\times \cancel{6} \times 5}{\cancel{3}\times\cancel{2}}\right)$

$= 105 + 126 + 35 = 266$ ways.

21. $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{\frac{10-r}{2}} \left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r \cdot x^{\frac{10-r}{2} - 2r} \cdot \left(\frac{1}{3}\right)^{\frac{10-r}{2} - r} \cdot \left(\frac{1}{2}\right)^r$$

For the term to be independent of x,

$$\frac{10-r}{2} - 2r = 0 \Rightarrow 10 - 5r = 0 \Rightarrow r = 2.$$

$$\therefore T_3 = T_{2+1} = {}^{10}C_2 \left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{2}\right)^2 = \frac{\cancel{10}^5 \times \cancel{9}}{\cancel{2}\times\cancel{9}\times 4} = \frac{5}{4}$$

(OR)

$${}^nC_1\, x^{n-1}\, y = 135 \rightarrow (1)$$

$${}^nC_2\, x^{n-2}\, y^2 = 30 \rightarrow (2)$$

$${}^nC_3\, x^{n-3}\, y^3 = \frac{10}{3} \rightarrow (3)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{n x^n y}{x} \times \frac{2x^2}{n(n-1)x^n y^2} = \frac{\overset{9}{\cancel{135}}}{\underset{2}{\cancel{30}}}$$

$$\Rightarrow \frac{x}{y} \times \frac{2}{n-1} = \frac{9}{2} \Rightarrow \frac{x}{y} = \frac{9(n-1)}{4} \rightarrow (4)$$

$$\frac{(2)}{(3)} \Rightarrow \frac{\cancel{n(n-1)}}{\cancel{2}} \times \frac{\cancel{x^n}}{x^2} \times \cancel{y^2} \times \frac{3\times \cancel{2}}{\cancel{n(n-1)}(n-2)} \times \frac{x^3}{\cancel{x^n}\, y^3} = \frac{3\cancel{0}\times 3}{1\cancel{0}}$$

$$\Rightarrow \frac{3}{n-2} \times \frac{x}{y} = 9 \Rightarrow \frac{3}{n-2} \times \frac{\cancel{9}(n-1)}{4} = \cancel{9} \text{ (from (4))}$$

$$\Rightarrow \frac{n-1}{n-2} = \frac{4}{3} \Rightarrow 3n-3 = 4n-8 \Rightarrow \underline{\underline{n=5}}$$

22. $3\times 8 + 6\times 11 + 9\times 14 + \dots$

$a_n = (n^{th}$ term of $3, 6, 9, \dots)(n^{th}$ term of $8, 11, 14, \dots)$

$= 3n(3n+5) = 9n^2 + 15n$

$\therefore S_n = \Sigma a_n = 9\Sigma n^2 + 15\Sigma n$

$$= \overset{3}{\cancel{9}} \frac{n(n+1)(2n+1)}{\underset{2}{\cancel{6}}} + 15\,\frac{n(n+1)}{2}$$

$$= \frac{3n(n+1)}{2}\left[2n+1+5\right]$$

$$= \frac{3n(n+1)}{2}(2n+6) = \underline{\underline{3n(n+1)(n+3)}}$$

23. 3 letters are chosen out of 9 letters in ${}^9C_3$ ways.

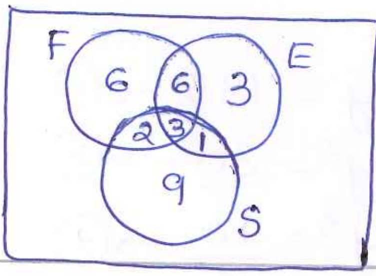
$$P\left(\text{more than one vowel will be selected}\right) = \frac{({}^4C_2 \times {}^5C_1) + {}^4C_3}{{}^9C_3}$$

$$= \frac{\left[\left(\frac{\overset{2}{\cancel{4}}\times 3\times 5}{\cancel{2}}\right) + 4\right]\times 6}{9\times 8\times 7}$$

$$= \frac{\overset{17}{\cancel{34}} \times \cancel{6}}{\underset{3}{\cancel{9}}\times\cancel{8}\times 7} = \frac{17}{42}$$

SECTION-D

24. (i) 2
(ii) 30
(iii) 20



25. $\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cdot \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$ or $2\cos 2x + 1 = 0$

$\Rightarrow 3x = n\pi$ or $\cos 2x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{3}, n \in Z$ $\qquad \Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$

(OR)

$$\frac{1-\cos x + \cos y - \cos(x+y)}{1+\cos x - \cos y - \cos(x+y)}$$

$$= \frac{(1-\cos x) + \left[-2\sin\left(\frac{x+2y}{2}\right)\cdot\sin\left(-\frac{x}{2}\right)\right]}{(1+\cos x) - \left[2\cos\left(\frac{x+2y}{2}\right)\cdot\cos\left(-\frac{x}{2}\right)\right]}$$

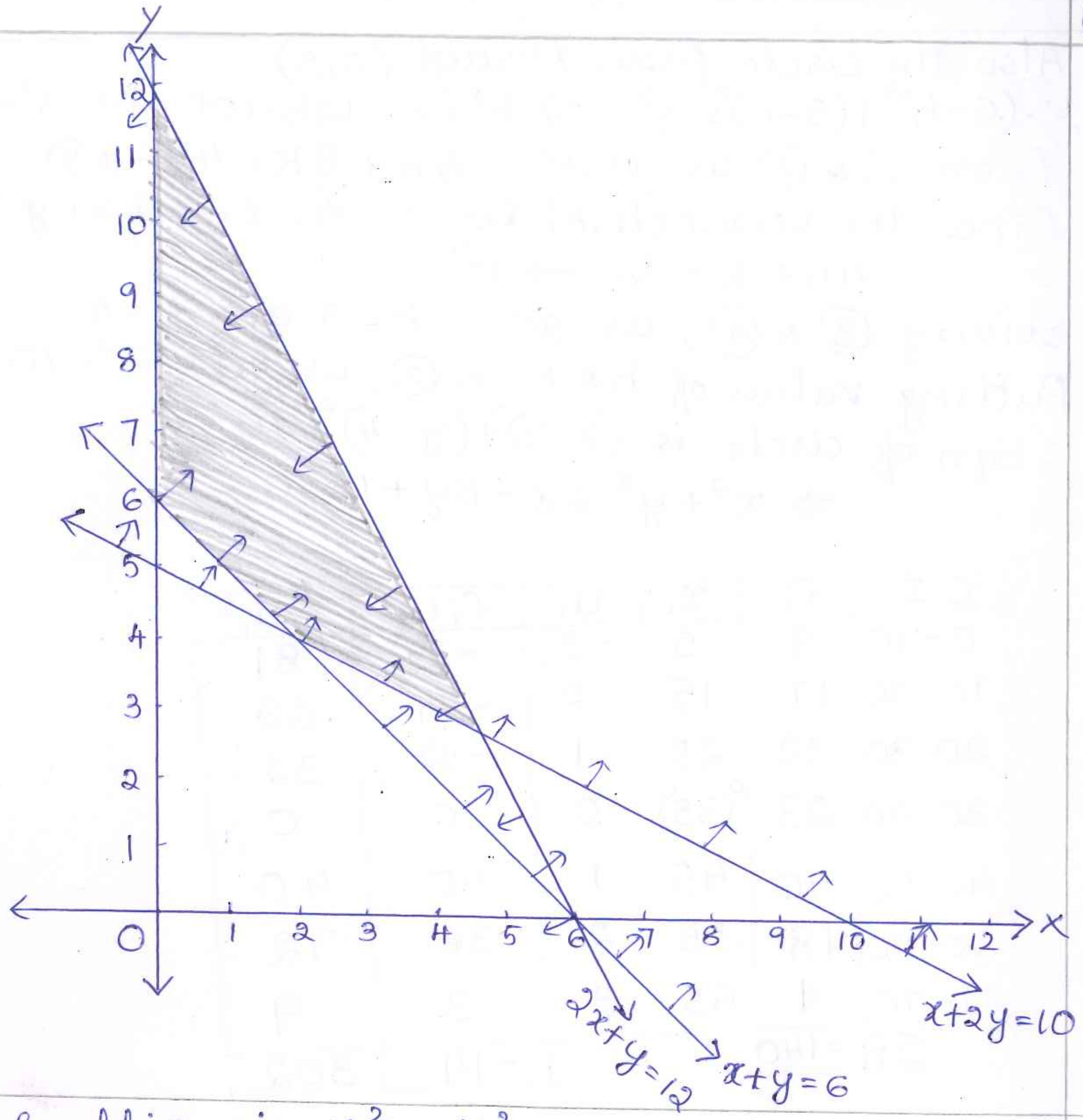
$$= \frac{(1-\cos x) + 2\sin\left(\frac{x+2y}{2}\right)\cdot\sin\left(\frac{x}{2}\right)}{(1+\cos x) - 2\cos\left(\frac{x+2y}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}$$

$$= \frac{2\sin^2\frac{x}{2} + 2\sin\left(\frac{x+2y}{2}\right)\cdot\sin\frac{x}{2}}{2\cos^2\frac{x}{2} - 2\cos\left(\frac{x+2y}{2}\right)\cdot\cos\frac{x}{2}}$$

$$= \frac{\cancel{2}\sin x/2\left[\sin x/2 + \sin\left(\frac{x+2y}{2}\right)\right]}{\cancel{2}\cos x/2\left[\cos x/2 - \cos\left(\frac{x+2y}{2}\right)\right]}$$

$$= \frac{\sin x/2\left[\cancel{2\sin(x+y)}\cdot\cos(-y/2)\right]}{\cos x/2\left[\cancel{+2\sin(x+y)}\cdot\sin(-y/2)\right]} = \tan\frac{x}{2}\cdot\cot\frac{y}{2}$$

26.



27. Eqn of ellipse is $\frac{x^2}{49}+\frac{y^2}{36}=1$

Now, $49>36 \Rightarrow a^2=49$ and $b^2=36$

$\therefore$ $a=7$, $b=6$, $c=\sqrt{a^2-b^2}=\sqrt{13}$

$\therefore$ coordinates of foci $=(\pm\sqrt{13},0)$

Coordinates of vertices $=(\pm 7,0)$

Length of major axis $=2a=2\times 7=14$

Length of minor axis $=2b=2\times 6=12$

Eccentricity $(e)=\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latusrectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

(OR)

Eqn of circle is $(x-h)^2+(y-k)^2=r^2$

Since the circle passes through $(4,1)$

$\therefore (4-h)^2+(1-k)^2=r^2 \Rightarrow h^2+k^2-8h-2k+17=r^2 \rightarrow (1)$

Also the circle passes through $(6,5)$

$\therefore (6-h)^2+(5-k)^2=r^2 \Rightarrow h^2+k^2-12h-10k+61=r^2 \rightarrow (2)$

From (1) & (2) we have, $4h+8k=44 \rightarrow (3)$

Since the centre (h,k) lies on the line $4x+y=16$,

$\therefore 4h+k=16 \rightarrow (4)$

Solving (3) & (4), we get, $h=3$ and $k=4$.

Putting values of h & k in (2), we get, $r^2=10$.

$\therefore$ Eqn of circle is $(x-3)^2+(y-4)^2=10$

$\Rightarrow x^2+y^2-6x-8y+15=0$

28.

C.I	f_i	x_i	u_i	f_iu_i	$f_iu_i^2$
0-10	9	5	-3	-27	81
10-20	17	15	-2	-34	68
20-30	32	25	-1	-32	32
30-40	23	a (35)	0	0	0
40-50	40	45	1	40	40
50-60	18	55	2	36	72
60-70	1	65	3	3	9
	$\Sigma f_i=140$			-14	302

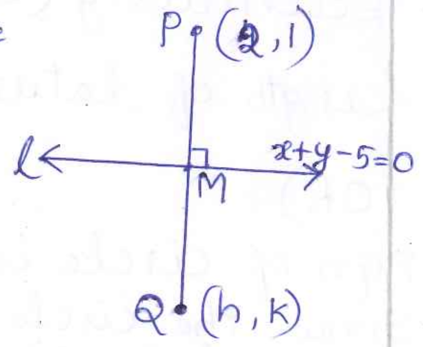
$$\bar{x} = 35+\frac{(-14)}{14\cancel{0}}\times 1\cancel{0} = 35-1=34$$

$$\sigma^2 = \frac{1\cancel{0}\times 1\cancel{0}}{14\cancel{0}\times 14\cancel{0}}\left[140\times 302-(-14)^2\right]$$

$$= \frac{\cancel{14}}{14\times\cancel{14}}\left[3020-14\right] = \frac{3006}{14} = 214.72$$

$$\sigma = \sqrt{214.72} = 14.65$$

29. Let $Q(h,k)$ be the image of the given point $P(2,1)$.



Slope of the line l is -1.

$\therefore$ Slope of $PQ = 1$

But slope of PQ is $\frac{k-1}{h-2}$

$\Rightarrow \frac{k-1}{h-2}=1 \Rightarrow h-k=1 \rightarrow (1)$

Let 'M' be the midpoint of PQ,

$\therefore$ Coordinates of $M = \left(\frac{2+h}{2}, \frac{1+k}{2}\right)$.

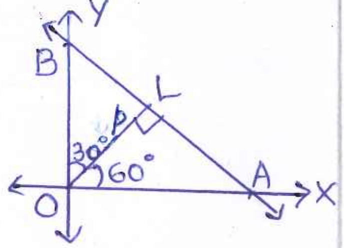
Now, 'M' lies on the line $x+y-5=0$

$\therefore \frac{2+h}{2} + \frac{1+k}{2} - 5 = 0 \Rightarrow h+k=7 \rightarrow$ (2)

solving (1) & (2) we get, $h=4$, $k=3$.

$\therefore$ Image of the point $(2,1)$ is $(4,3)$.

(OR)



Let AB be the given line & $OL = p$ be the perpendicular.

Given, $\alpha = 60°$

$\therefore$ Eqn of AB is $x\cos 60° + y\sin 60° = p$

$$\Rightarrow \frac{x}{2} + \frac{\sqrt{3}y}{2} = p \Rightarrow x + \sqrt{3}y = 2p \rightarrow (1)$$

In ΔOLA, $\cos 60° = \frac{OL}{OA} \Rightarrow \frac{1}{2} = \frac{p}{OA} \Rightarrow OA = 2p$

In ΔOLB, $\cos 30° = \frac{OL}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{p}{OB} \Rightarrow OB = \frac{2p}{\sqrt{3}}$

It is given that area of $\Delta OAB = 54\sqrt{3}$ sq. units

$\therefore \frac{1}{2} \times OA \times OB = 54\sqrt{3}$

$\Rightarrow \frac{1}{\cancel{2}} \times \cancel{2}p \times \frac{2p}{\sqrt{3}} = 54\sqrt{3}$

$\Rightarrow p^2 = 81 \Rightarrow p = 9$

Substituting the value of p in (1), we get,

$x + \sqrt{3}y = 18$.

This is the required equation of line AB.